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Patterns appear in physical, chemical, and biological systems and are characteristically striking and reproducible. Consequently, scientists and mathematicians have developed theories to explain the origins of these patterns. There are several approaches to the study of pattern; ours is based on symmetry and bifurcation.

We have investigated both the theory and application of symmetric dynamical systems and its relation to pattern formation. In this work, 'pattern' is identified with invariance under some of the underlying symmetries. In bounded domains, we have investigated bifurcations to spiral waves. We have also investigated chaotic dynamics where the pattern appears as symmetry on average.

In a separate direction, we have considered pattern formation in unbounded domains. Our theoretical investigations have included the bifurcation and meandering of spirals and the Ginzburg-Landau theory of spatially extended systems — with application to spatially aperiodic solutions in spatially extended systems.

Finally, part of our effort has been devoted to studying the dynamics present in (ordinary) differential equations with symmetry. For example, we have studied stable ergodicity of chaotic attractors in problems with continuous symmetry, and the existence, stability and bifurcations of robust heteroclinic cycles. Some of these ideas are relevant to intermittent magnetic dynamos in rotating Rayleigh-Bénard convection.

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Final Report

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Dynamics, Symmetry and PDEs

2/15/94 — 11/15/96

by

Martin Golubitsky, Michael Field and Ian Melbourne

We describe briefly the results obtained with ONR support. The twenty-five papers that acknowledge ONR support under Grant N00014-94-1-0317 (or will when completed) are given in the reference section.

1. The symmetry of attractors

As predicted in our previous work, the symmetry of attractors translates physically to patterns in the time average of experiments that are not present in instantaneous images. This prediction has been confirmed in the experiments on the Faraday surface wave model by Gollub's group at Haverford College. For example, in both square and circular geometries, Gollub's group found states with full symmetry.

Solutions with less symmetry are theoretically possible in differential equations. When the underlying group of symmetries is finite, we have a complete theoretical understanding of the possible symmetry types of attractors. The possible symmetry types were previously classified for (noninvertible) continuous maps; we have classified the possible symmetry types for diffeomorphisms and flows [13]. (See [12] for an overview.) In particular, when the dimension of the state space is large enough, every subgroup of symmetries can arise in a structurally stable manner as the symmetry group of an attractor. This result is relevant for PDEs which have infinite dimensional state spaces.

The problems that arise when the symmetry group is compact but infinite are of a completely different nature. It is now possible to perturb along continuous group orbits to show that certain subgroups arise as the symmetry group of an attractor only in degenerate situations. In particular, it is often the case that the symmetry group of an attractor includes all the continuous symmetries. Aspects of this issue have been studied in [11, 25, 6].

An interesting mathematical question concerns determining the symmetry of attractors in high dimensional spaces. Previously we discovered a numerical algorithm for making this determination when the symmetry group is finite. Research to improve this algorithm is now complete [17, 15] and we have begun the extension to continuous groups [7].

2. Spiral waves

Although spiral waves have been studied in numerous contexts, there are virtually no existing bifurcation results that lead to spiral waves from a homogeneous equilibrium. We have shown [5]— both theoretically and computationally — that spiral waves can result from *spiral* boundary conditions in a single reaction diffusion equation. This result is surprising since it is widely believed that spiral patterns can only occur in systems of PDE. However, this belief is based on the assumption of Neumann or Dirichlet boundary conditions which leads to selfadjoint linear problems.

In a different direction, Barkley noted that bifurcation from spiral waves to meandering waves (a Hopf bifurcation) can lead to a resonance situation where the spiral tip meanders off to infinity in a specified direction. (This observation has been confirmed both numerically and in chemically reacting systems.) Barkley also noted that this resonance appears to be a consequence of Euclidean symmetry and this phenomenolgical observation has been proved rigorously by the Berlin group (Wulff, Sandstede, Scheel, Fiedler). In [16] we noted specifically how the phase space analysis of the Berlin group manifests itself in physical space and the fact that special features of this type of Hopf bifurcation occur on bifurcation from many armed spirals.

The whole question of the drift along group orbits associated with relative equilibria and relative periodic orbits can be studied abstractly for noncompact groups. This has been done in [1].

3. Heteroclinic Cycles

Heteroclinic cycles are models for intermittency in solutions of differential equations and are known only to occur generically in systems of equations with special structure — in particular they are known to occur in systems with symmetry. Lumley and

Holmes have used this type of dynamics to model intermittency in certain boundary layer problems.

Heteroclinic cycles appear in symmetric systems both by spontaneous symmetry breaking and by forced symmetry breaking. Sufficient, usually necessary, conditions for the asymptotic stability of cycles are given in [20, 21]. We have found a method for creating many examples of heteroclinic cycles in certain symmetric systems of ODEs called coupled cell systems [9]. A low dimensional example of a heteroclinic cycle obtained through forced symmetry breaking is presented in [19]. Secondary bifurcation *from* heteroclinic cycles is discussed in [3, 21]. (The analysis of this bifurcation is motivated by the magnetic dynamo problem in geophysics [3].) See also [9].

We have also discovered that intermittency and cycles can occur connecting dynamically complex (even chaotic) attractors. This has led to the phenomenon of cycling chaos [4, 9].

4. Pattern Formation and Symmetry Breaking Bifurcations

There is an intimate relation between symmetry breaking bifurcations where solutions with less symmetry and more pattern appear by bifurcation from a group invariant homogeneous equilibrium. We have studied a variety of these bifurcations during the past two years — both from the point of view of theory and application. Most of this research involves compact symmetry groups ([8, 9])

The most studied bifurcations are to solutions with maximal isotropy. Interesting examples of maximal isotropy subgroups are given in [22] and discussions of solutions with less than maximal isotropy are discussed in [8, 9]. Results on symmetry breaking for maps (rather than differential equations) are given in [10].

Specific kinds of pattern that we have studied include: new patterns in Euclidean invariant systems [2], patterns that appear in discretizations of reaction-diffusion equations with Neumann boundary conditions on a square [14], and the work on spiral waves mentioned previously. We have also studied the symmetry constraints on solutions to mechanical systems with two degrees of symmetry [18].

5. Ginzburg-Landau Equations

Local bifurcations in spatially-extended systems are supposed to be governed by 'universal' amplitude or modulation equations known as the *Ginzburg-Landau equations.* In [23, 24] we consider systems of partial differential equations equivariant under the Euclidean group and undergoing steady-state bifurcation (with nonzero critical wavenumber) from a fully symmetric equilibrium. A rigorous reduction procedure is presented that leads locally to an optimally small system of equations. When there are one or two unbounded spatial directions, reduction leads to a single equation.

In analogy with equivariant bifurcation theory for compact groups, we give a classification of the different types of reduced systems in terms of the absolutely irreducible unitary representations of the Euclidean group. In particular, we prove that the Ginzburg-Landau equation on the line is indeed universal. Furthermore, we show in [23] how to derive amplitude equations. Standard truncations of these amplitude equations are known to be constant coefficient and odd. This structure is a consequence of 'normal form' symmetry [23] and is not present to all orders.

In the plane, there are precisely two significantly different types of reduced equation: *scalar* and *pseudoscalar.* These equations are known to have very different solutions and dynamics [2]. For example, the standard planforms such as rolls and hexagons are replaced in the pseudoscalar case by anti-rolls and oriented hexagons.

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